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A. A. BREITKREUTZ

THE POWER FACTOR OF A CONDENSER AND A GENERAL METHOD FOR MEASURING IT.

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THE POWER FACTOR OF A CONDENSER AND A GENERAL METHOD FOR MEASURING IT

by

ADELINE ALBERTINE BREITKREUTZ

A Thesis Submitted for the Degree of
BACHELOR OF ARTS

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1908

6882336

In an alternating electrostatic field, energy is consumed in the dielectric medium in what is called electrostatic or dielectric hysteresis. Thus when an alternating electromotive force is impressed on a capacity, the current through it is not ninety degrees out of phase with this E.M.F., but less than this, and therefore not wattless.

Let B be the angle of lead of the current through the imperfect condenser, over the impressed E.M.F. Then $\cos B = \sin A$ is the power factor of the condenser, where angle A equals ninety degrees minus angle B.

An equivalent loss of energy can be produced by substituting for the imperfect condenser of capacity C, a perfect condenser of exactly the same capacity C, with a resistance p in series with it (Fig. 1). Then

$$\tan A = wCp \qquad (1)$$

where w equals 2π times the frequency n. The same power factor is obtained by putting a resistance W in parallel with the perfect condenser C (Fig. 2). Then

$$\tan A = \frac{1}{wCW} \qquad (2)$$

In order that wCp shall equal $\frac{1}{wCW}$, we see that W must be large and p small.

For good, well insulated air condensers, the power factor is neglible, but for poorly constructed condensers, paper condensers, and even for good mica condensers, it is appreciable.

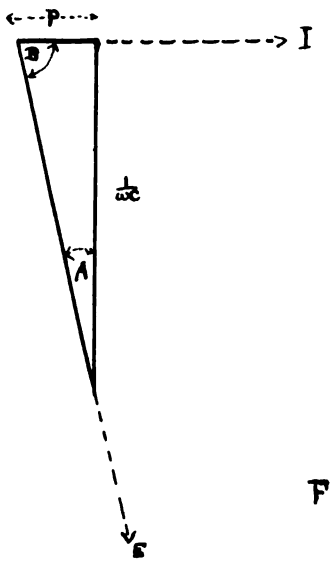

Fig. 1.

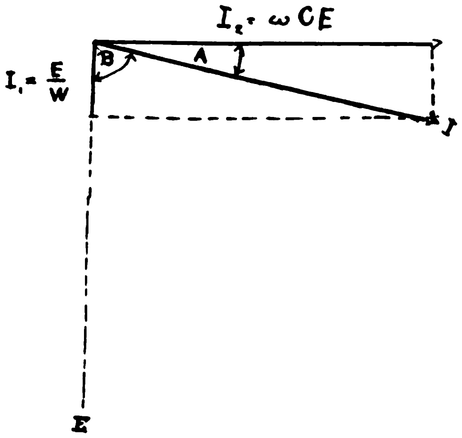

Fig. 2.

This loss of power may be considered as a constant loss of energy per cycle, for a given frequency and E.M.F. The loss of energy per cycle varies with the frequency.

Absorption effects become less as a higher frequency is used, since they depend on the time rate of charge and discharge of the condenser. They are due to the discharge of the "soak," where the percentage of the residual charge which is discharged is a function of the time. Since there is a direct relation between the power factor and the soak of a condenser, we would expect the former as well as the latter to be largest for low frequencies, and to decrease as the frequency increased, the rate of decrease with increase of frequency depending on the particular condenser used.

The frequency may be made so great as to permit of practically no discharge of the soak. The effective capacity is then smaller at a high frequency than it is at a lower one. The change of capacity due to temperature is troublesome in methods of measuring cos B which depend on the value of the capacity, for when the power factor is large, the testing current appreciably heats the condenser.

The power factor has been measured with a fair degree of accuracy by various methods. One class of methods consists in measuring the energy loss, and from this computing cos B. This has been done by the use of a watt-meter, or

by measuring the amount of heat generated in the condenser. But the energy loss is very small, and therefore difficult to measure.[1]

Another line of procedure has been to get directly the phase relation of condenser current to the impressed E.M.F. This has been done by various methods.

VARIOUS FORMS OF ALTERNATING CURRENT BRIDGES WITH TWO BALANCE CONDITIONS.[2] The first of the forms used consists of two non-inductive resistances, R_3 and R_4 for two arms of the bridge, and a capacity with a resistance in series with it for each of the other two (Fig. 3.). In this case, it has been found most convenient to represent the soak by a resistance p in series with the capacity. Either a telephone or a vibration galvanometer may be used across AB.

The impedances of the different arms are,

$Z_3 = R_3$; $\qquad Z_1 = -jc_1 + p_1 + r_1$;

$Z_4 = R_4$; $\qquad Z_2 = -jc_2 + p_2 + r_2$;

where $c_1 = \frac{1}{wC_1}$ and $c_2 = \frac{1}{wC_2}$

Then for a balance,

$$\frac{R_3}{R_4} = \frac{-jc_1 + p_1 + r_1}{-jc_2 + p_2 + r_2}$$

From which it follows,

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[1] Rosa and Smith, Phys. Rev., Jan. and Feb., 1899.

[2] Grover, Bulletin Bureau of Standards, Aug., 1907.

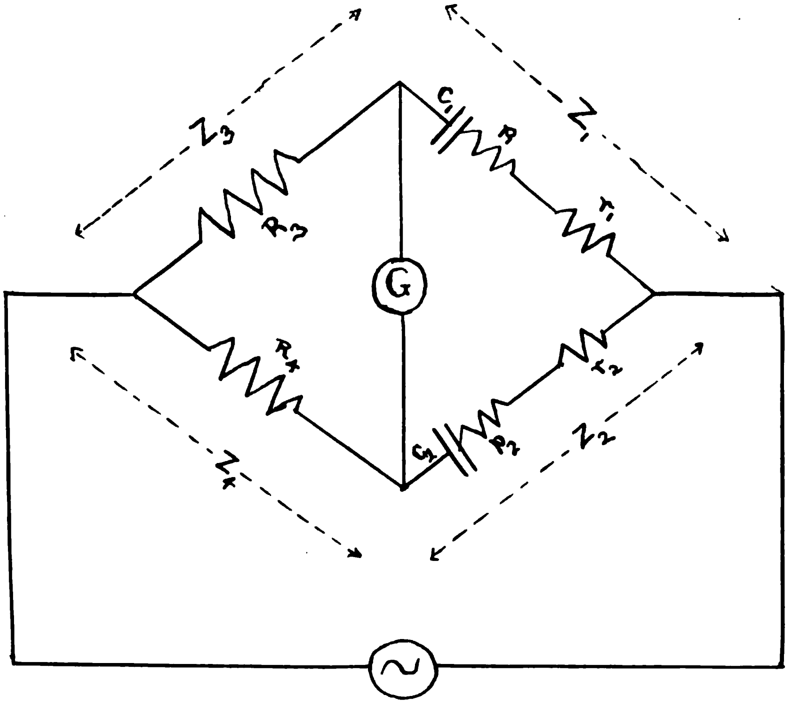

Fig. 3.

$$\frac{p_2 + r_2}{p_1 + r_1} = \frac{R_4}{R_3} \qquad (3)$$

and

$$\frac{C_1}{C_2} = \frac{R_4}{R_3} \qquad (4)$$

Also

$$\frac{p_2 + r_2}{p_1 + r_1} = \frac{C_1}{C_2} \qquad (5)$$

From (5), $wC_2p_2 - wC_1p_1 = wC_1r_1 - wC_2r_2$

or $\tan A_2 - \tan A_1 = wC_1r_1 - wC_2r_2$

Writing $\tan A_2 - \tan A_1 = \tan(A_2 - A_1)$ as an allowable approximation, we have,

$$\tan(A_2 - A_1) = wC_1r_1 - wC_2r_2 \qquad (6)$$

It follows from equations (4) and (6) that the ratio of two condensers and the difference in their phase angles, $(A_2 - A_1)$, can be obtained simultaneously on a bridge of this kind.

Sources of error are(1) distributed capacity or inductance in R_3 and R_4; (2)in $r_1$and r_2, and (3) electrostatic induction between the bridge and its surroundings, which errors become of importance when the two power factors to be compared are nearly equal, or the capacities are under one tenth of a microfarad.

A simple method, called the substitution method, for

eliminating electrostatic capacity, and largely also the other errors when the two capacities and their power factors do not differ very much, is as follows;

Let C_2 be an auxiliary condenser, and C_1 and C_1' the two condensers for which we wish to find $(A_1' - A_1)$. We first balance for C_1, then substitute C_1' for C_1 and rebalance by altering R_3 to R_3' and r_1 to r_1'. Then

$$\frac{C_1'}{C_1} = \frac{R_3'}{R_3}$$

and

$$\tan(A_1 - A_1') = wC_1'\left(r_1' - \frac{R_3'}{R_3}\cdot r_1\right) \qquad (8)$$

The only error which effects one balance and not the other is the induction of $(R_1' - R_1)$. This is small when C_1 and C_1' are almost alike. The method even as a substitution method is thus limited.

Fig.4 gives a diagram for a modification of the same form of bridge. L_3 and L_4 are inductances one of which is continuously variable. R3 is the total resistance of the arm AB, part of it being a variable non inductive resistance, and similarly for R_4. For a balance, the following equations must hold,-

$$\tan(A_1 - A_2) = \frac{wL_3}{R_3} - \frac{wL_4}{R_4} \qquad (9)$$

$$\frac{C_1}{C_2} = \frac{R_4}{R_3}\left(1 + \frac{wL_3}{R_3}\tan A_2 - \frac{wL_4}{R_4}\tan A_1\right) \qquad (10)$$

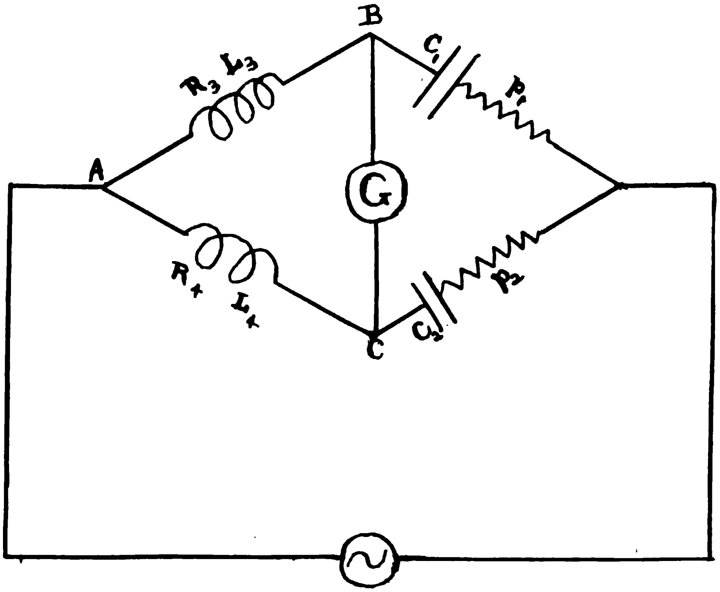

Fig. 4

and we again obtain simultaneously $\frac{C_1}{C_2}$ and $(A_1 - A_2)$. The correction factor in equation (10) is usually negligible.

Here, as in the other case, the substitution method is to be recommended. If C_2 is the auxiliary condenser, and C_1 and C_1', the two condensers to be compared, then

$$\tan(A_1 - A_1') = \frac{wL_3}{R_3} - \frac{wL_3'}{R_3'} \qquad (11)$$

$$\frac{C_1}{C_1'} = \frac{R_3'}{R_3}\left[1 + \tan(A_1 - A_1')\left\{\tan A_2 - \frac{wL_4}{R_4}\right\}\right] \qquad (12)$$

Fig.5 illustrates a method of using the conception of soak as equivalent to a resistance in parallel with the capacity. W_1 and W_2 represent the equivalent resistances of soak, in parallel with C_1 and C_2. Let k_1 and k_2 be the distributed capacity of R_1 and R_2 which are rather large resistances. Then we may derive the following balance conditions,-

$$\frac{C_1}{C_2} = \frac{R_4}{R_3}\left[1 + \left(\frac{k_2}{C_2} - \frac{k_1}{C_1}\right)\right] \qquad (13)$$

$$\tan(A_1 - A_2) = \frac{1}{wC_2R_2} - \frac{1}{wC_1R_1} \qquad (14)$$

If the power factors of C_1 and C_2 are nearly equal, the

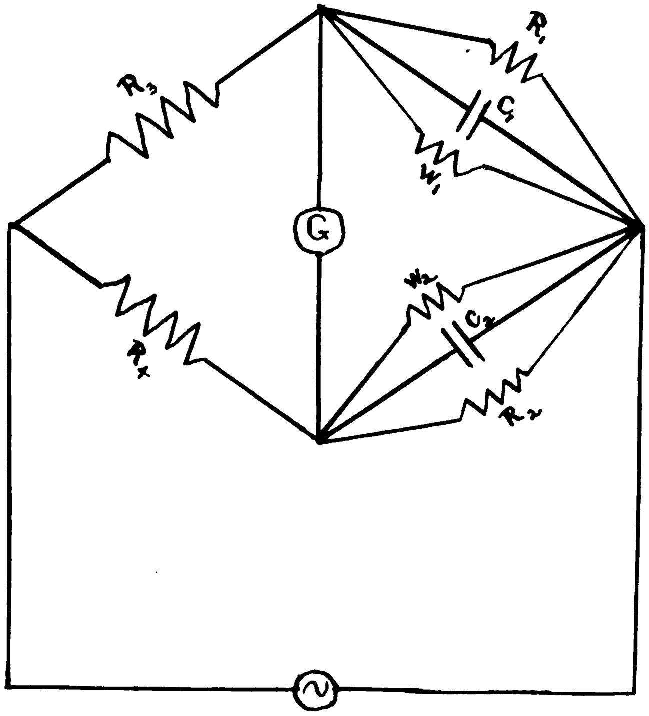

Fig. 5

correction factor in (13) becomes negligible. As a substitution method, we get, using the same nomenclature as in the previous cases,-

$$\tan (A_1 - A_1') = \frac{1}{wC_1'R_1'} - \frac{1}{wC_1R_1} \qquad (15)$$

and

$$\frac{C_1}{C_1'} = \frac{R_3'}{R_3}\left[1 + \frac{k_1'}{C_1'} - \frac{k_1}{C_1}\right] \qquad (16)$$

where the correction factor in (16) is usually negligible.

The differential transformer can be used to get $(A_1 - A_2)$, if C_1 and C_2 have been previously determined by some other method. L_1 and L_2 (Fig.6) are two coils wound on the same iron core, of exactly the same induction, and of an ohmic resistance R_1 and R_2, as nearly alike as possible. A telephone is in series with the secondary, which is not shown on the diagram. r_1 is the total resistance outside of R_1 and W, in one branch, and r_2 in the other. C_1 and C_2 are first short circuited and W, an ohmic resistance, made infinity. A balance is then obtained by adjusting the slide wire. Then the currents in L_1 and L_2 are exactly equal and opposite, and in phase. The capacities C_1 and C_2 are now put in and the balance readjusted by shunting the coil in series with the larger capacity by a resistance W. The currents through L_1 and L_2 are again the same in

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1. Trowbridge, Phys. Rev., Feb. 1905.

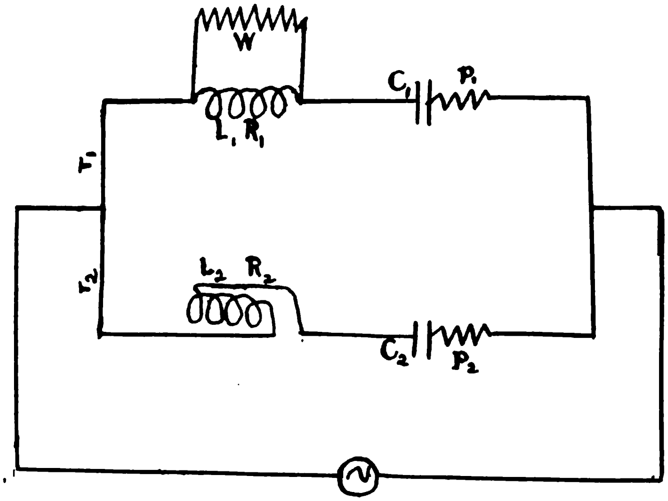

Fig. 6.

in phase and magnitude. We can then equate the tangents of their phase angles.

$$\frac{1}{wC_2(R_2 + r_2 + p_2)} = \frac{1}{wC_1\left(r_1 + p_1 + \frac{R_1W}{R_1 + W}\right)}$$

or

$$\tan(A_2 - A_1) = w\left[C_1\left(r_1 + \frac{R_1W}{R_1 + W}\right) - C_2(r_2 + R_2)\right] \qquad (17)$$

The distributed capacity of only one box W must be accounted for. All other errors are symmetrical, unless C_1 and C_2 are so unlike that the difference in current through the two branches for the second balance causes, on account of difference in heating effect, a difference in resistances R_1 and R_2 which did not exist in the preliminary balance. But for two nearly equal capacities, this method ought to be applicable.

For small capacities, the series inductance method is the most reliable of those described, because there is no capacity outside of C_1 and C_2 in the arms containing the condensers to be tested. The transformer and the parallel resistance methods have not the sensibility of the series inductance or series resistance methods.

Determination of the Absolute Value of the Power

Factor.-- If we have a condenser for which A or p are known at a number of temperatures and frequencies, then we can, of course, get the power factor for another condenser of the same nominal value by any of the three substitution methods. Condenser leads must be corrected for. Or, with a well insulated air condenser, the resistances of whose leads ~~small~~ are small enough to leave A essentially zero, we can determine the power factor of a condenser of the same nominal value in the same way. But air condensers are usually no greater than .03 M.F. A "stepping-up" process is then used. The angles for several small capacities are determined directly by comparison with the air condenser. They are then joined in parallel, and the joint angle is equal to the weighted mean of the separate angles.

A method for determining the power factor absolutely is given by Anderson's Bridge, applicable where no air condenser or standard is at hand. P, R, S, and r are non inductive variable resistances (Fig. 7). Q is the resistance of a variable inductance coil L.

Considering the power factor as a resistance p in series with C, the following conditions hold for a balance;

$$Pi_1 + r(i_2 + ji'_2) = (Q + jx)(i4 + ji'_4) \qquad (18)$$

$$R\,(i_3 + ji'_3) = (r + p - jc)(i_2 + ji'_2) \qquad (19)$$

$$S\,(i_4 + ji_4') = (p + jc)(i_2 + ji_2') \qquad (20)$$

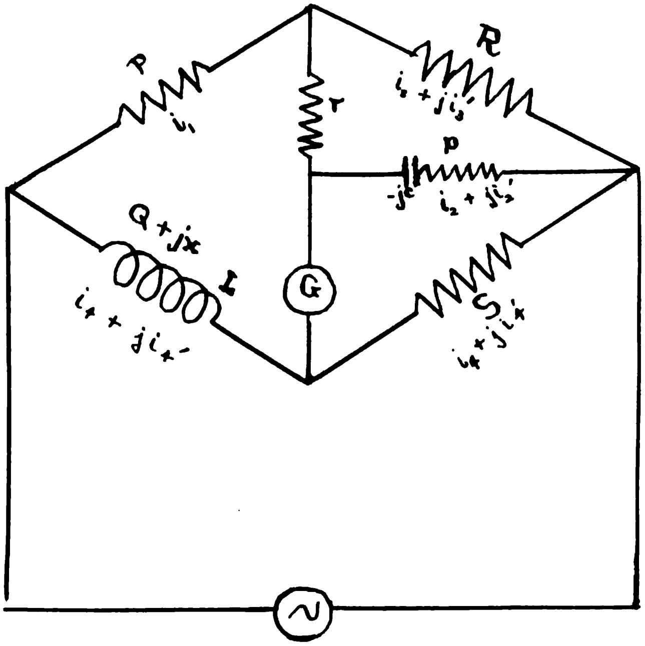

Fig. 7.

$$i_1 = i_3 + ji'_3 + i_2 + ji'_2 \tag{21}$$

where the currents are as shown in the diagram. On eliminating the current from these equations, we find that the following conditions must hold,[1]-

$$\frac{PS(R + r + p)}{R} + Sr - pQ = \frac{L}{C} \tag{22}$$

$$PQ - SP = Rpw^2LC \tag{23}$$

When $P = R$, (22) and (23) become

$$S(R + 2r + p) - pQ = \frac{L}{C} \tag{24}$$

$$Q - S = pw^2LC \tag{25}$$

which can be written in the form

$$L = CS\left[2r + P(1 - \tan^2 A)\right] \tag{26}$$

$$Q = S + wL\tan Q \tag{27}$$

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[1] The substitution method here gives the following equations, when $P = R$,-

Equations (22) and (23) show that this form of bridge is impossible for the ordinary commercial alternating current when the condenser has absorption. The steady current balance differs from the alternating current balance and this, as shown by equation (23), varies as the square of the frequency. The commercial alternating current is not uniform enough to permit of considering w as a constant. The higher harmonics, which act like several frequencies superposed, also make a balance impossible for a bridge whose balance depends on w . The ratio $\frac{L}{C}$ is also affected by the power factor to a considerable extent. Hence the method is altogether undersirable for use with a commercial alternating current. The object of this paper is to describe a form of bridge independent of frequency which is applicable as a means of finding p absolutely without the use of either air condenser or standard.

$$\frac{C'}{C} = \frac{2r_1 + P}{2r_1' + P}(1 + [\tan^2 A' - \tan^2 A])$$

$$Q' - Q = wL \tan (A' - A)$$

When used as such, it ceases to give an independent determination of A for one condenser. Since A enters into the equation for $\frac{C'}{C}$, our determination of this ratio depends on having a constant known w.

The form of bridge given above can be so modified that the balance is independent of w. Fig.8 shows how this may be done by shunting the inductance with a noninductive resistance. Although this form of bridge and the one next given have not been tried experimentally, there is every reason to suppose that both would prove practical. The general principles involved are, as far as can be seen, the same as those which apply to the bridge shown in Fig.11, which has been used with success.

Balance conditions (Fig.8) are;

$$Pi_1 + r(i_2 + ji_2') = \frac{(Q + jx)m}{Q + jx + m}(i_4 + ji_4')$$

$$R(i_3 + ji_3') = (r + p + jc)(i_2 + ji_2')$$

$$S(i_4 + ji_4') = (p - jc)(i_2 + ji_2')$$

$$i_3 + ji_3' = i_1 - (i_2 + ji_2')$$

from which we derive on eliminating current, the following relations;

$$\frac{P(m + Q)}{Qm} = \frac{R}{S} \qquad (28)$$

$$p = \frac{S[R(P + r) + Pr]}{Rm - PS} \qquad (29)$$

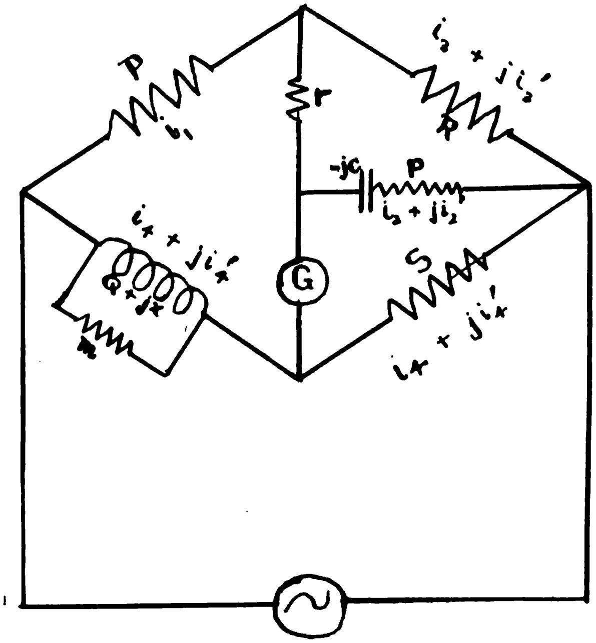

Fig. 8.

$$\frac{L}{C} = \frac{QS\left[(P + r)R + Pr\right]}{Rm - PS}$$

$$\frac{L}{C} = (Q + m)p \qquad (30)$$

If m is large, change in the resistance of the inductance coil due to its temperature coefficient does not introduce a large error. These equations are considerably simplified when $P = R$.

A form of the same bridge which gives a simpler equation for p is shown in Fig. 9. Then for a balance,

$$\frac{r_1(r_2 + p - jc)}{r_2(p - jc)} = \frac{m(r_3 + jx)}{(r_3 + m + jx)r_4}$$

from which it follows for a bridge independent of w,-

$$\frac{r_1(r_3 + m)}{r_3 m} = \frac{r_2}{r_4}$$

$$p = \frac{r_1 r_2 r_4}{m r_2 - r_1 r_4}$$

$$\frac{L}{C} = \frac{(r_3 + m)(r_2 + p)r_1 r_4 - m r_3 r_2 p}{m r_2 - r_1 r_4}$$

When $r_1 = r_2$,

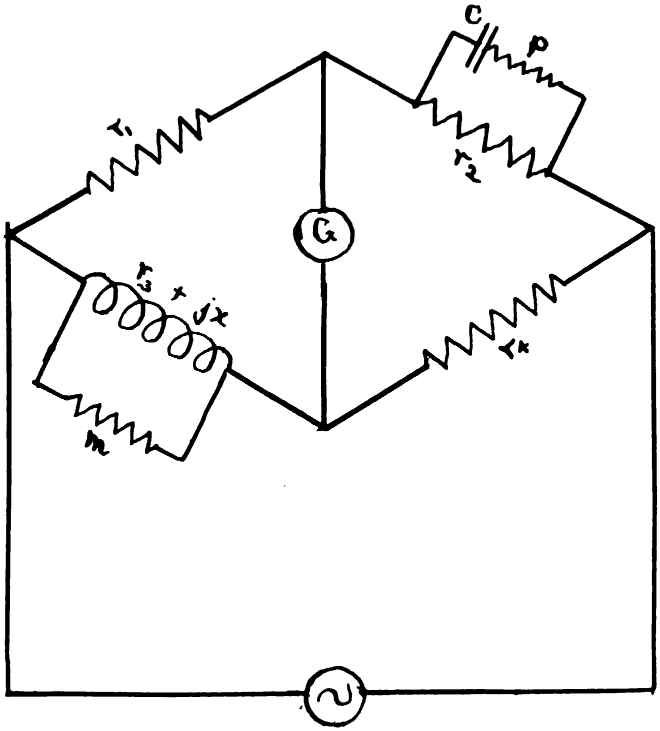

Fig. 9.

$$r_4 = \frac{r_3 m}{r_3 + m} \qquad (31)$$

$$p = \frac{r_1 r_4}{m - r_4} \qquad (32)$$

$$\frac{L}{C} = \frac{(r_3 + m)(r_1 + p)r_4 - mr_3 p}{m - r_4}$$

or $$\frac{L}{C} = \frac{r_1 r_3 m}{m - r_4} \qquad (33)$$

Equations (31), (32), and (33) are all simple. (32) gives an equation for p independent of both L and C, and also of r_3, the resistance of the induction coil.

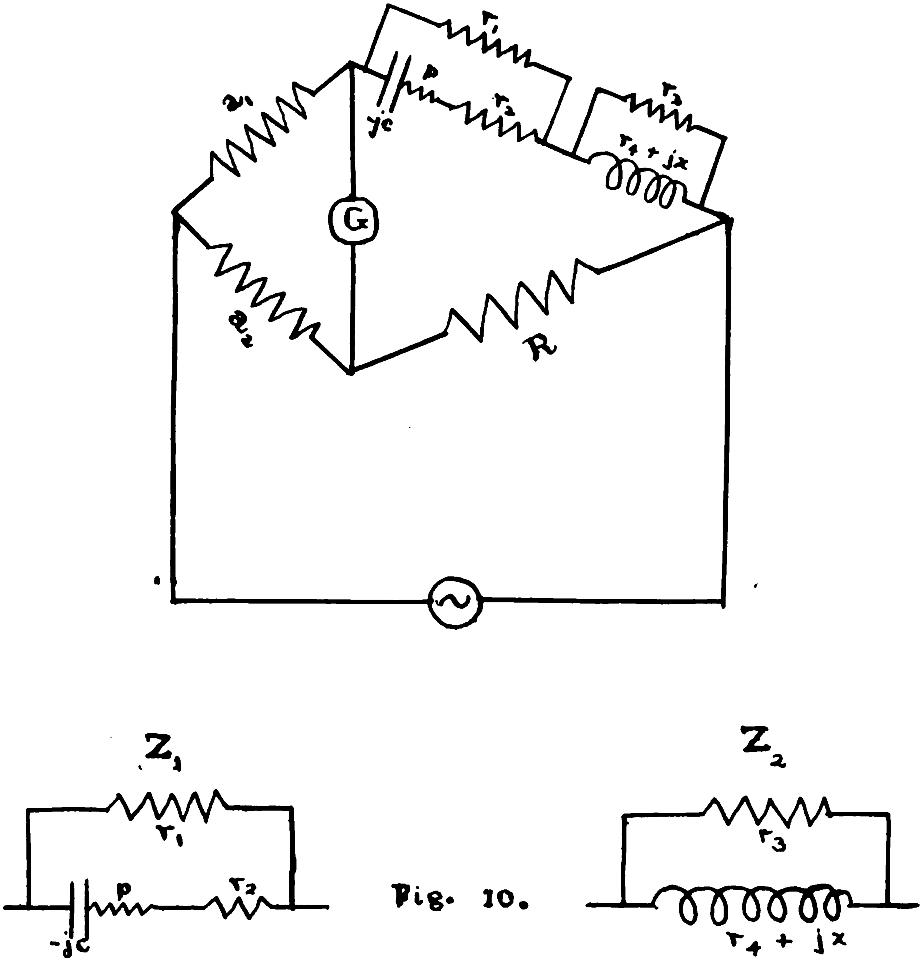

Fig. 10.

A BRIDGE WITH CAPACITY AND INDUCTANCE IN THE SAME ARM SUITABLE FOR MEASURING THE POWER FACTOR ABSOLUTELY WITH A COMMERCIAL CURRENT.[1] The form of bridge actually used for determining p is diagramed in Fig. 10. a_1, a_2, r_1, r_2, r_3 are non-inductive resistances. $a_1 = a_2$. r_4 is the resistance of a variable induction coil L.

Derivation of Formulae. Let Z equal impedance of the arm containing L and C. Then

$$\frac{a_1}{a_2} = \frac{Z}{R} = 1 \qquad \text{or} \qquad Z = R$$

$$Z = Z_1 + Z_2$$ where Z_1 and Z_2 are as indicated in Fig. 10.

$$\frac{1}{Z_1} = \frac{1}{r_1} + \frac{1}{(r_2 + p) \;\; -jc}$$

$$\frac{1}{Z_2} = \frac{1}{r_3} + \frac{1}{(r_4 + jx)}$$

Then
$$Z_1 = \frac{r_1(r_2 + p) - r_1 jc}{(r_2 + p) \;\; -jc + r_1}$$

and
$$Z_2 = \frac{r_3 r_4 + r_3 jx}{r_3 + jx + r_4}$$

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1. Taylor, Phys. Rev., May, 1907.

From the above, we can write,

$$R = \frac{r_1(r_2 + p) - r_1 jc}{(r_2 + p) - jc + r_1} + \frac{r_3 r_4 + r_3 jx}{r_3 + r_4 + jx}$$

This may be expanded as follows,

$$\left| \begin{array}{l} R(r_2 + p)r_4 + Rr_1 r_4 - Rr_4 jc + \\ R(r_2 + p)r_3 + Rr_1 r_3 - Rr_3 jc + \\ R(r_2 + p)jx + Rr_1 jx + Rxc \end{array} \right| =$$

$$\left| \begin{array}{l} r_1(r_2 + p)r_4 - r_1 r_4 jc + \\ r_1(r_2 + p)r_3 - r_1 r_3 jc + \\ r_1(r_2 + p)jx + r_1 xc \end{array} \right| + \left| \begin{array}{l} (r_2 + p)r_3 r_4 - r_1 r_3 r_4 - r_3 r_4 jc + \\ (r_2 + p)r_3 jx + r_1 r_3 jx + r_3 xc \end{array} \right|$$

Putting the imaginaries equal to zero,-

$$\left| \begin{array}{l} - Rr_4 jc + Rr_3 jc \\ + R(r_2 + p)jx + Rr_1 jx \end{array} \right| = \left| \begin{array}{l} - r_1 r_4 jc \qquad - r_1 r_3 jc \\ + r_1(r_2 + p)jx - jcr_3 r_4 \\ + r_3(r_2 + p)jx + r_1 r_3 jx \end{array} \right|$$

But $jx = wL$ and $jc = \frac{1}{wC}$

Then on simplifying,

$$- Rr_4 - Rr_3 + R(r_2 + p)w^2LC + Rr_1w^2LC = - r_1r_4 - r_1r_3 + r_1(r_2 + p)w^2L$$

For the bridge to be independent of frequency, we must have separately,

$$Rr_4 + Rr_3 = r_1r_4 + r_1r_3 + r_3r_4$$

and $$R(r_2 + p) + Rr_1 = r_1(r_2 + p) + (r_2 + p)r_3 + r_1r_3$$

or $$R = r_1 + \frac{r_3r_4}{r_3 + r_4} \qquad (34)$$

and $$\frac{r_1^2}{r_3^2} = \frac{r_1 + (r_2 + p)}{r_3 + r_4} \qquad (35)$$

From the reals,-

$$R\left[(r_2 + p)(r_3 + r_4) + r_1(r_3 + r_4)\right] + Rxc = r_1(r_2 + p)(r_3 + r_4) + \left[(r_2 + p) + r_1\right]r_3r_4 + r_3xc$$

Substituting for R,

$$R = r + \frac{r_3r_4}{r_3 + r_4}$$

and simplifying and combining terms,

$$\frac{L}{C} = \frac{r_1^2}{r_3^2}(r_3 + r_4)^2$$

But

$$\frac{r_1^2}{r_3^2} = \frac{r_1 + (r_2 + p)}{r_3 + r_4}$$

Then

$$C = \frac{L}{\left[r_1 + (r_2 + p)\right](r_3 + r_4)} \qquad (36)$$

From equations (34), (35) and (36), it is possible to find p without knowing L and C or their ratio, or the frequency n. $\frac{L}{C}$ and p can be determined simultaneously.

General Discussion of the Apparatus.--- Fig. 11 is a diagram of the bridge as it was used. a_1 and a_2 are two resistances made of an alloy wire with a negligible temperature coefficient, and wound noninductively. One was measured on a potentiometer, and then the other substituted for the first, and so adjusted that it balanced on the same setting. The two can then be relied on to differ by no more than two parts in a hundred thousand. They introduce no error of distributed capacity or inductance, because they enter in exactly the same manner into both sides of the bridge. For R , r_1 , r_2, and r_3, Wolf resistance boxes were used. They were standardized on a direct reading potentiometer which is correct to one part in ten thousand. Since most of the values used for R, r_1, r_2, r_3 are under 150 ohms, the largest resistance coil used was of magnitude

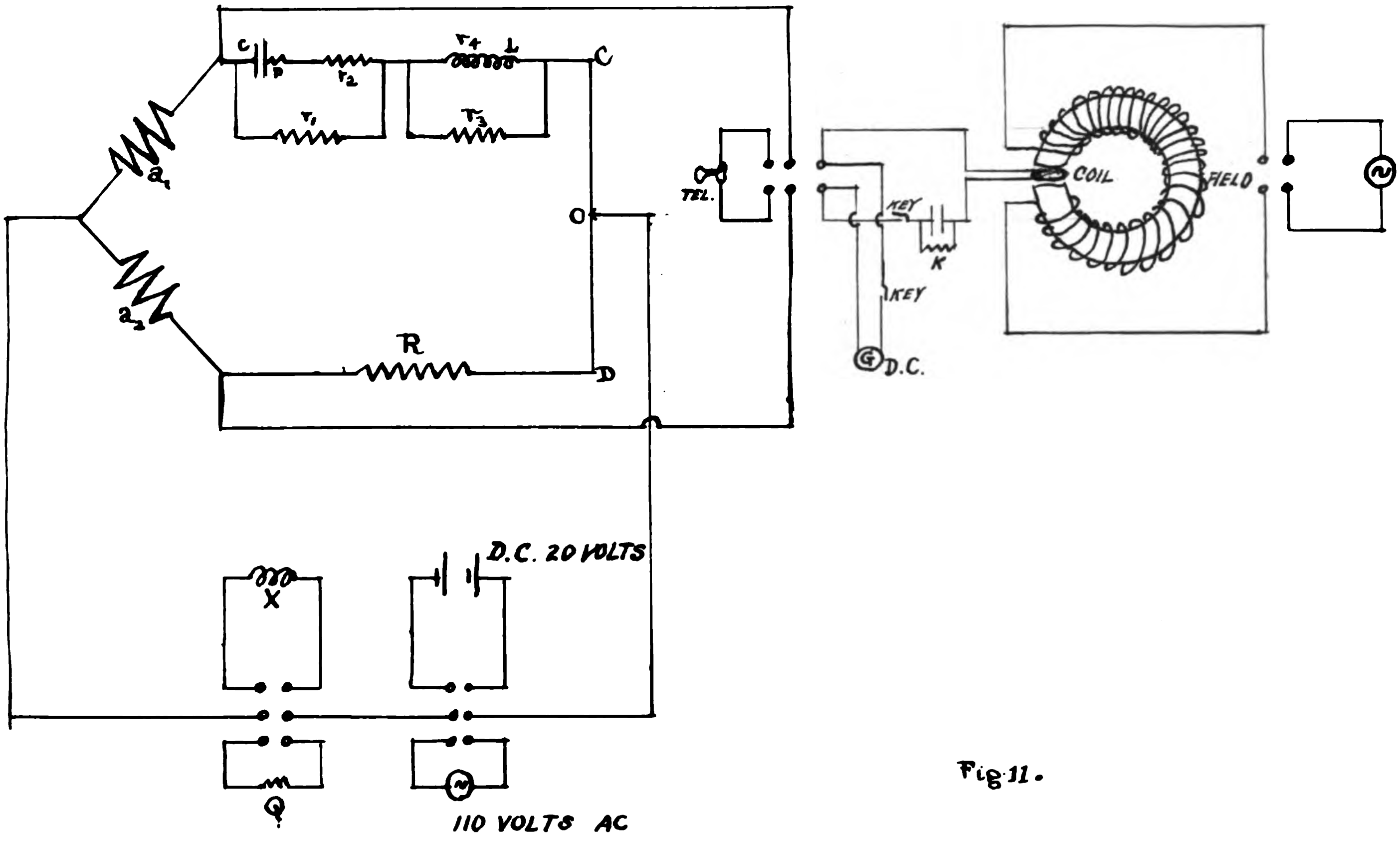
c
p
r_2
r_4
L
C
r_1
r_3
a_1
a_2
O
R
D
TEL.
KEY
K
KEY
G
D.C.
COIL
FIELD
D.C. 20 VOLTS
X
Q
110 VOLTS AC

Fig 11.

can be put in series with it. When a very small capacity is to be measured, r_1 and r_3 can be taken of such magnitude that the resulting L is large enough to minimize error due to distributed capacity and inductance. The standard of inductance is put at a distance of about eighteen feet from the rest of the set-up, in order that its stray field may not react with the rest of the bridge. Its lead wires are of low resistance, wound non-inductively. The inductance itself is of copper wire, and has a high temperature coefficient. R, r_1 and r_3 do not vary sensibly with temperature or frequency, Since it is difficult to determine the temperature of r_4 exactly, its resistance was computed for each balance from equation (34). (This is impossible in the series inductance method given by Grover, because there we have inductance coils in two arms of the bridge.)

All lead wires, and where possible, contact resistances were measured and taken into account. The lead wires were wound non-inductively and made as short as possible to minimize the effect of stray inductance to a negligible amount.

A double-throw switch is placed between A and B, Fig. 11. Thrown over one way, it puts a telephone across A and B, and, in the other, the coil of an alternating current galvonometer. A vibration galvanometer could not be used,

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1. Taylor, Phys. Rev., July, 1907.

because the current to be detected is of inconstant frequency and ragged wave-form. The field of the galvanometer is exited by the same source of E.M.F. as the bridge. The alternating current galvanometer could also be used as a direct current galvanometer by sending a direct, instead of an alternating current through the field. This was found impracticable in this case, however, as the galvanometer had to be placed so far away from the rest of the set-up on account of its stray field, that the difference in the two zero points necessitated a longer scale than could be conveniently used. The coil is shunted by a control K, consisting of a capacity in parallel with a suitable resistance, to obtain the proper sensibility.[1] The rest of the control is furnished by the bridge itself (Fig. 12).

There are two double throw switches in the external circuit. By means of one of them, we can apply either an alternating or a direct E.M.F. The latter was used merely as a means of finding out how much the direct differed from the alternating current balance point. With the two condensers used, this was found to change R by less than five parts in a hundred thousand. This difference is accounted for by the increase in resistance of r_4 with alternating over direct current. Leakage can also be detected by comparing the two direct current balances obtained respectively with

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[1] Taylor, Phys. Rev., July, 1907.

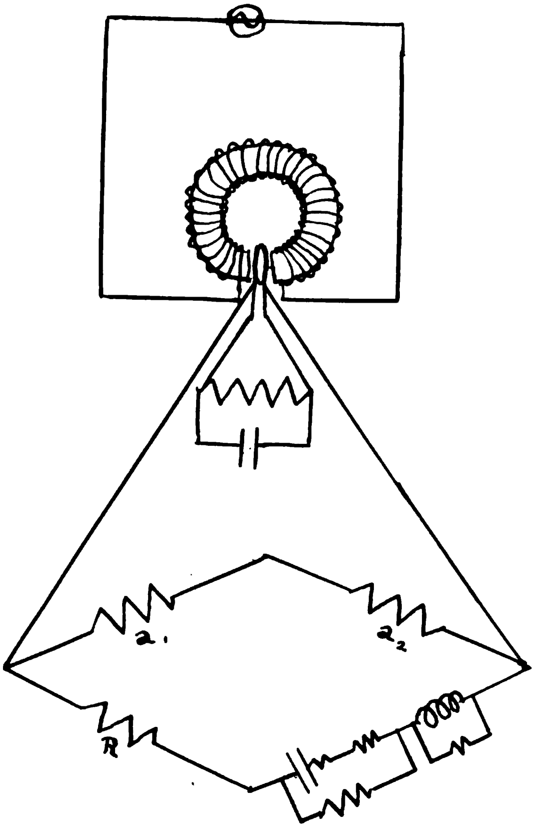

Fig 12.

the capacity in closed and in open circuit, although this is not the same for both direct and alternating current balance.

An alternating current balance was first obtained with the second switch in the external circuit so closed that a non-inductive resistance Q (Fig. 11) was placed in series with the bridge. The telephone was first used to get an approximate balance. This was completed with the alternating current galvanometer.

With the resistance in series with the bridge, the current to be detected has a certain phase angle with respect to the field. This phase angle is changed when the switch is thrown over to put an inductance in the external circuit. If there is zero deflection in the one case because the current to be detected and the field are at right angles, then we cannot get zero deflection due to this cause when the current angle is changed. The bridge is so adjusted that there is no deflection for both phases of the external current. The balance must then be a true one.

Sources of Error. It has already been shown that distributed capacity and inductance of the resistances produce no appreciable error; neither does the temperature coefficient of r_4. The only other error which remains outside of error of reading settings of slide wires, is the electrostatic capacity of the entire bridge with respect to earth. It was found, as far as could be judged, that a sound in the

telephone of the same intensity as that produced by electrostatic capacity could be overcome by a shift of about half a centimeter on the slide wire DC of Fig. 11. This corresponds to a change in R of about four parts in a hundred thousand.

The effect of electrostatis capacity is also easily observed on the alternating current galvanometer. With the switch at B (Fig. 13) open, opening and closing the bridge key gives a deflection of such magnitude as to confirm the conclusions already reached with the telephone. With key A closed, and B open, there is a surging of potential with the alternations through the whole bridge, and consequently the same electrostatic capacity of bridge with respect to earth which we have with key B closed, except in so far as the introduction of X or Q (Fig. 11), both at a distance from the bridge, change it. It was found that for a balance, there is zero deflection with B open, or thrown on either X or Q, which would seem to indicate that our balance on the alternating current galvanometer does not depend on electrostatic capacity at all.

Application of Substitution Method. To get $(p_1 - p_2)$ for condensers C_1 and C_2 is a simple matter. We first obtain a balance with C_1. Then we replace C_1 by C_2, and rebalance by means of L and r_2. The only errors then introduced are errors of setting r_2 and r_2'. $r_2 - r_2' = p_1 - p_2$ is easily set to .01 ohms where r_2 is small, and can be read as close-

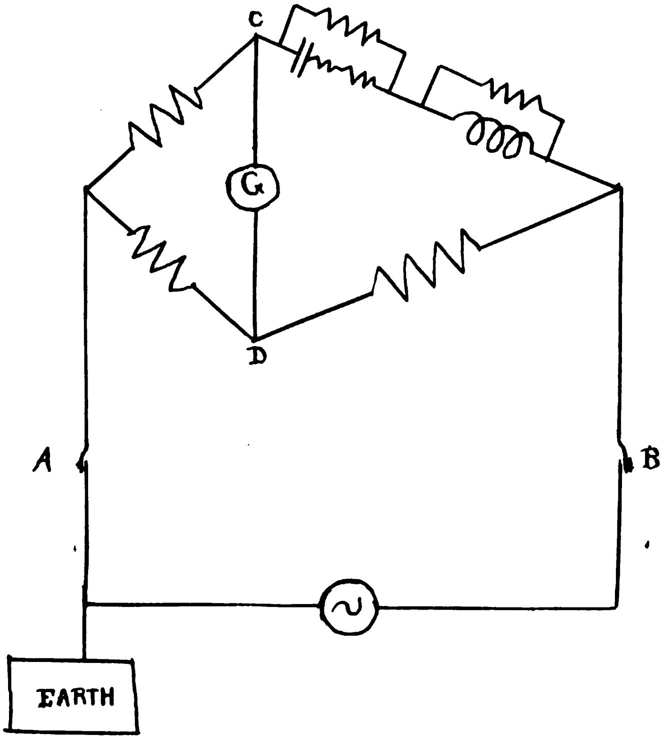

Fig. 13.

ly as set on a properly calibrated slide wire. A telephone is as free from error as a golvanometer in this instance, except in so far as it is less sensitive, and presents the advantage of requiring less time in the using. C_1 and C_2 may be unlike without introducing error due to change in electrostatic capacity or distributed capacity and inductance, for the total effect of that arm of the bridge containing L and C must be the same for both balances. Thus we have here an advantage of this as a substitution method over the others previously discussed.

General Considerations. We notice several interesting facts by analyzing equations (34), (25), and (36). In the first place, equation (35), used for computing p, is independent of frequency, except in so far as p varies with it. The data show that this variation is not of importance with the variation of frequency found in the commercial alternating current.

The bridge affords two methods for determining $\frac{C_1}{C_2}$ which serve as a check on each other, as well as a method for getting C_1 in terms of L. One means for finding $\frac{C_1}{C_2}$ with great accuracy is extremely simple. A balance with C_1 is first obtained. Then C_1 is replaced by C_2 and another balance secured by adjusting L and r_2. For

$$C_1 = \frac{L}{(r_1 + r_2 + p_1)(r_3 + r_4)}$$

and

$$C_2 = \frac{L + \Delta L}{(r_1 + r_2' + p_2)(r_3 + r_4)}$$

whence since

$$r_2 + p_1 = r_2' + p_2$$

we have

$$\frac{C_1}{C_2} = \frac{L}{L + \Delta L}$$

This method allows us to correct for absorption in getting the ratio of two capacities without actually finding the magnitude of the absorption. It is applicable where a well calibrated standard inductance is at hand, the only possibility of error being in reading L. An inductance calibrated to tenths of a degree, will permit of reading ordinary inductances to one part in a thousand, and the setting can be made closer than this. The inductance used was calibrated with a commercial current like that here used. Therefore its value as read does not differ from its real value due to change of inductance with frequency. Neither p_1 or p_2, or any of the resistances need be known. When a balance is impossible by merely adjusting L and r_2, it can be assumed that the difficulty lies in a change in the resistance of the inductance coil, for which correction can be made by

means of a slide wire at the end of the inductance lead.

After having found p_1 and p_2, we can get C_1/C_2 by leaving L unchanged for both balances, and readjusting r_2, r_1 and r_3 for the second balance. The first method is the better one when C_1 and C_2 are very unlike. When we have two condensers of the same nominal value, the second is perhaps preferable. In applying the second method, it is best to take r_2 large enough in both cases to make the effect of error in the computed p_1 and p_2 negligible. A capacity can also be computed in terms of the inductance, using equation (36). Here again, it is adviseable to compute p from a setting where r_2 is small, and then to use that value in the above equation, taking a new balance with r_2 large.

In the practical use of the bridge, when it is desired to obtain p absolutely, error is minimized, as before indicated, by taking r_2 so small that p is about one-tenth of it at the least. The setting of r_2 then becomes extremely sensitive. In order that r_2 be small, the following conditions must hold:

$$r_1(r_3 + r_4) > r_3^2, \text{ but } r_1(r_3 + r_4) \doteq r_3^2.$$

Also $R = r_1 + \frac{r_3 r_4}{r_3 + r_4}$, from which we can get approximate values of R, r_1, r_3 to satisfy these conditions.

$(r_1 + r_2)(r_3 + r_4)$ must at the same time be kept with-

in certain limits to correspond with the range of variation of the inductance. It is not necessary to make a special correction for the leads of the capacity. C and r_2 are short circuited, and leads and contact resistances measured on the potentiometer. They then form a part of r_2.

The variations in p as found by this method give an outstanding difference of five parts in eight hundred, which is less than six tenths per cent.

The inductance in series with the bridge makes the telephone balance less liable to error resulting from high frequency harmonics which would otherwise be magnified by the capacity. It was found that with a capacity in series with the bridge, one balance was obtained with the telephone and another with the galvanometer. The inductance has the effect of weakening the higher harmonics enormously, and was used for that reason.

The Data. The first table gives the results obtained with a telephone condenser, using a telephone in order to obtain the balance. The second table gives results for the same condenser where the alternating current galvanometer was used as a current detector. The second set gives results which do not differ as much among themselves as do those obtained with the use of the telephone. In table three, is given a set of results for an Elliot Mica Condenser. The frequency of the commercial alternating current was sixty

cycles per second, which gives values for A which for the order of the magnitude measured, compare favorably as to the consistency of the several determinations for the same quantity, with the values obtained by Grover.

The method was also tried as a substitution method for obtaining $(p_1 - p_2)$ for the two condensers used, and gave very consistent results, as was expected. The ratio C_1/C_2 could not be worked out as accurately as the settings were made, because the standard of inductance was not calibrated finely enough to permit of settings being read as closely as they were made.

TABLE I.

C = 2.01 M.F.

r_1	r_2	r_3	R	p
Ohms	Ohms	Ohms	Ohms	Ohms
94.640	3.713	99.055	103.419	0.860
111.621	4.899	115.0725	120.515	0.835
78.564	10.640	77.055	86.9733	0.821
128.516	3.632	133.074	137.375	0.826

Average p = 0.835 Ohms.

Tan A = 0.0006327

A = 4.45"

TABLE II.

C = 2.01 M.F.

r_1	r_2	r_3	R	p
Ohms	Ohms	Ohms	Ohms	Ohms
51.027	7.448	52.054	59.1586	0.832
111.456	4.679	115.067	120.324	0.826
94.6304	3.654	99.055	103.336	0.830
77.444	9.123	77.054	85.8758	0.833
128.606	3.847	133.0726	137.590	0.829

Average p = 0.831 Ohms.

Tan A = 0.0006297

A = 4.32"

TABLE III.

C = 1.00 M.F.

r_1	r_2	r_3	R	p
Ohms	Ohms	Ohms	Ohms	Ohms
128.646	4.825	133.073	137.690	0.023
94.180	4.061	99.055	102.968	0.021
76.804	9.199	77.054	85.285	0.019
111.3808	5.407	115.067	120.245	0.020
188.045	5.931	191.074	197.095	0.012
73.324	3.003	79.054	81.9508	0.017
72.819	1.407	80.056	81.454	0.017
107.961	.740	116.067	116.827	0.022
107.946	1.841	115.067	116.892	0.024
87.270	1.951	94.056	95.885	0.018
87.300	.705	95.056	95.770	0.016

Tan A = 0.0000071638

A = 0.049255"

Average p = 0.019 Ohms.

Approved A. Hoyt Taylor

Asst. Prof. of Physics

Dated June 9. 1908.

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